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**Time-Symmetric Time Steps:**

**Block Time Steps**

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# Preface

## 0.1 xxx

We thank xxx, xxx, and xxx for their comments on the manuscript.  
Piet Hut and Jun Makino



# Chapter 1

## Introduction

### 1.1 preliminary summary

In `world6.rb` we present an individual time step code, with an option to use block time steps, and another option to perform the block time steps in a time reversible way.

Time symmetric block time step algorithms were first introduced in *Time-Symmetric Block Time-Step Algorithm for N-Body Simulations*<sup>1</sup>, by Makino, J., Hut, P., Kaplan, M. & Saygin, H., 2006, *New Astronomy* **12**, 124-133. We have extended their procedure, presented in their paper only for the leapfrog algorithm, to a general approach that is applicable to all symmetric integration schemes (symmetric for constant time steps).

For example, the leapfrog integration scheme is explicitly symmetric in time for constant time steps. It was show in *Building a Better Leapfrog*<sup>2</sup>, by Hut, P., Makino, J. & McMillan, S., 1995, *Astrophys. J. Lett.* **443**, L93-L96, how to use an iteration scheme to construct a time-symmetric version of the leapfrog, even for adaptive time steps, with step length changing in time, but still using the same shared time step for all particles, at any given point in time. The paper by Makino et al, quoted above, extended that analysis to an individual time step scheme, based on block time steps, where each time step length is a fraction of the fundamental time step length, multiplied by a negative power of two.

In `world6.rb` we offer a code that combines block time steps and time reversability not only for the leapfrog, but for a variety of other schemes, including the standard fourth-order Hermite scheme, as well as a sixth-order Hermite scheme. Currently, the fourth-order Hermite scheme is explicitly time symmetric, and

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<sup>1</sup><http://arXiv.org/abs/astro-ph/0604371>

<sup>2</sup>[http://adsbit.harvard.edu/cgi-bin/nph-iarticle\\_query?bibcode=1995ApJ...443L..93H](http://adsbit.harvard.edu/cgi-bin/nph-iarticle_query?bibcode=1995ApJ...443L..93H)

passes the time reversability test (integrating forwards, and then backwards for the same duration), reaching the starting point to within machine accuracy. The sixth-order Hermite does not come that close, probably because our interpolation scheme is not quite fully symmetric.

nil nil nil nil nil nil nil nil nil



## Chapter 2

# Literature References

[to be provided]